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Special Session in Logic and Foundations of Mathmematics
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Keynote Speaker

Yiannis N. Moschovakis (University of California, Los Angeles and National & Kapodistrian University of Athens)

The mathematics of definitions

Definability theory is an important part of logic and (especially) its applications, cutting across the traditional subdivisions of the field into Model theory, Proof theory, Set theory and Computability. In most cases, however, a serious attempt to study the objects which are *definable* in some specific way leads inevitably to a study of the *definitions* we accept: for a trivial example, to study the *computable functions* on the natural numbers, we must understand Turing machines (or some other *equivalent* computation model), which in turn brings up interesting problems not easily formulated or solved in terms of the functions that are computed, e.g., questions of complexity.

My aim in this talk is to examine whether some properties of systems of definitions can be formulated abstractly and then used to establish results about the definable objects which cannot (easily or at all) be proved directly. My emphasis will be on examples, some of them from Descriptive Set Theory, in which Lebesgue first identified the importance of studying definable functions (on the real numbers) in a classical 1905 paper. It will be an elementary, mostly expository talk, and I will assume only some knowledge of logic and Turing computability.

Invited Speakers

Vassilios Gregoriades (University of Turin)

Uniformity functions in descriptive set theory and their applications

Suppose that X, Y are non-empty sets and that $P \subseteq X \times Y$ satisfies the property that for all $x \in X$ there is some $y \in Y$ such that $(x, y) \in P$. From the Axiom of Choice one can obtain a *choice-function* $u : X \to Y$ such that for all $x \in X$ we have $(x, u(x)) \in P$. We are concerned with the question of finding a "definable" choice-function. Here our underlying spaces are Polish, i.e. the topological spaces that arise from complete separable metric spaces. We give two examples of choice-functions, in the first the choice-function is Borel-measurable, and in the second it is continuous. Both these examples have important consequences in seemingly unrelated topics. The former is joint work with Takayuki Kihara and Keng Meng Ng.

About the Borel-measurable choice-functions. In many naturally occurring cases of Borel sets $P \subseteq \mathcal{X} \times \mathcal{Y}$ it is possible to find Borel-measurable choice-functions. These include the cases where for all $x \in \mathcal{X}$ we have that: P_x is (a) compact, or (b) countable, or (c) non-meager, or (d) of positive measure (for some fixed σ-finite Borel-measure). The area of *effective descriptive set theory* explains the underlying cause for the validity of these results. This boils down to the fact that we can find points in the sections P_x, which are "definable from x". More specifically there is a canonical assignment $\tilde{d} : \mathcal{X} \to \mathcal{X}^\omega$ such that for each x, $\tilde{d}(x)$ is a sequence that enumerates all points that are "definable from x" (the $\Delta^1_1(x)$-points). Moreover this assignment is done in a Borel-way, so that if P is Borel and for all $x \in \mathcal{X}$, $P_x \cap \tilde{d}(x) \neq \emptyset$ then there is a Borel-measurable choice-function for P, namely $u(x) = \tilde{d}(x)(n_x)$, where n_x is the least index of a term in $\tilde{d}(x)$ that belongs to P_x. If P_x satisfies one of the preceding $(a) - (d)$ then $P_x \cap \tilde{d}(x) \neq \emptyset$. (This is a consequence of well-known theorems in effective descriptive set theory.) So from the preceding P admits a Borel-measurable choice-function.

About the continuous choice-functions. Unlike the preceding method, where the choice-functions are established by the existence of definable points, *continuous* choice-functions are usually obtained by a direct construction, typically using well-founded trees and *bar recursion*. The standard example is the *Souslin-Kleene Theorem* which establishes the existence of a continuous function that realizes the separation property of the analytic sets. Our above mentioned example of a continuous choice-function is a Souslin-Kleene-type result based on a separation theorem by Preiss.

Contains joint work with Takayuki Kihara and Keng Meng Ng

Antonis Kakas (University of Cyprus)

Argumentation Logic

Argumentation Logic is born out of the growing pressure in Artificial Intelligence (AI) to develop human-like systems which can have a symbiotic relationship with their users. Personal or Cognitive Assistants are required to operate typically with a Natural Language interface and to possess cognitive or thinking faculties that are common in the natural intelligence of people. Such systems presuppose that we are able to sufficiently formalize the human form of common sense reasoning and decision making into a logical system.

Motivated by work in AI one approach to develop a framework for this type of informal logical reasoning is to base this on argumentation. In its most abstract form an argumentation framework in AI is defined as a tuple $< Arg, Att >$ where, Arg, is a set of arguments and Att is a binary (partial) relation on Arg, called the *attacking relation* on Arg. The central semantical notion of argumentation, namely that of a valid or acceptable argument, is given by formally capturing the statement: **"An argument is acceptable iff it renders all its attacking arguments (i.e. its counter-arguments) not acceptable"**. To do so we consider the following recursive operator of acceptability:

Let $AF =< Arg, Att >$ be an abstract argumentation framework and $\mathcal{R}$ the set of binary relations on 2^{Arg}. Then the **acceptability operator**, $\mathcal{F} : \mathcal{R} \to \mathcal{R}$, is defined as follows. For any $acc \in \mathcal{R}$ and $\Delta, \Delta_0 \in 2^{Arg}$:

$\mathcal{F}(acc)(\Delta, \Delta_0)$ iff

- $\Delta \subseteq \Delta_0$, OR,
- For any A such that A attacks Δ,
  - $A \not\subseteq \Delta_0 \cup \Delta$, AND
  - there exists D that attacks A such that $acc(D, \Delta_0 \cup \Delta \cup A)$.

This operator is monotonic w.r.t. set inclusion and hence its repeated application starting from the empty binary relation has a least fixed point.

Argumentation Logic (AL) is then concerned with the realization of argumentation frameworks and their least fixed point semantics. For example we can reconstruct classical Propositional Logic as such an AL realization but in a way that does not trivialize under inconsistent premises.

This work is based on jint work with P. Mancarella and F. Toni and helpful discussions with Vassilis Gregoriades.

Panagis Karazeris (University of Patras)

Conceptual completeness in categorical logic

We explain the correspondence between certain classes of small categories (with particular properties), on the one hand, and of certain classes of first-order theories (with particular syntactic complexity), on the other. We focus on regular and coherent theories. Regular theories consist of sequents $\varphi \vdash_{\vec{x}} \psi$, where φ, ψ are built from atomic formulae by $\wedge$ and $\exists$. Coherent theories allow further the use of $\vee$ in the formation of formulae. The latter have the same expressive power as full first-order logic, if we allow appropriate modifications of language. The algebraic counterpart of a regular theory is that of a regular category, i.e one with finite limits and regular epi - mono factorizations (sufficient for expressing $\exists$) that are stable under pullback ($\exists$ is compatible with substitution of terms).

Under the above correspondence, models of regular theories are just regular functors to the category of sets, i.e functors that preserve finite limits and regular epis. Models of a theory are now organized as a category. Its objects are regular functors $F\colon \mathcal{C} \to \mathrm{Sets}$ and its morphisms are natural transformations between them (which amount to homomorphisms between models). An interpretation of theories $F\colon \mathcal{C} \to \mathcal{D}$ induces by restriction a functor between the respective categories of models. A natural question is: If the induced functor between the categories of models is an equivalence, is F an equivalence as well? The straight answer is no. But it induces an equivalence at the level of *effectivizations* of the respective regular categories. Effectivization is the process of universally turning a regular category into an effective (=Barr-exact) one, i.e making every equivalence relation the kernel pair of its coequalizer. The completed category of a syntactical category $\mathcal{C}_{\mathbb{T}}$ of a theory $\mathbb{T}$ is nothing else than the syntactical category of the theory $\mathbb{T}^{eq}$, introduced by S. Shelah (exactly by adding new sorts for quotients of definable equivalence relations).

Such results (conceptual completeness) were first proved by M. Makkai and G. Reyes using model-theoretic arguments. Later A. Pitts gave, for coherent theories, a categorical proof valid over any base topos with a natural number object, when allowing models to take values in a *sufficient class of toposes.* A purely categorical, intuitionistically valid argument, for the case of regular theories, was given in joint work of the author with V. Aravantinos-Sotiropoulos. The improved intuitionistic version of conceptual completeness can also be of use: For rings R, S inside a topos (sheaves of rings, in plain terms) that are internally coherent, the theories of flat modules are regular (internal) theories. Equivalence of their (indexed) categories of flat modules yields an equivalence mod-$R \simeq$ mod-S of (internal categories of internally) finitely presentable modules. This might simplify rather complicated situations studied in Algebraic Geometry.

Pavlos Peppas (University of Patras)

Belief revision: achievements and challenges

More that three decades ago, a new research area was born at the crossroads of Formal Philosophy, Cognitive Science, and Artificial Intelligence. This area in now known as *Belief Revision.* It studies the process by which a rational agent changes her beliefs in the light of new information. For example, suppose that Margarita, an archaeologist, discovers ancient Greek coins during the excavation of an ancient tomb in Japan. This will change Margarita's beliefs on ancient trade routes and ancient Japan's seafaring technology, or even on the wider impact of classical Greece to the Far East. On the other hand, the discovery is not likely to have any effect on Margarita's views on social benefits or on combat-sports. Such belief change scenarios (albeit not as dramatic) are common when an intelligent agent is interacting with her environment. Belief revision is a central cognitive capability, and thus its modelling is important in a number of disciplines. What makes the modelling problem non-trivial is that, in principle, it is not enough to simply add the new information to one's stock of beliefs; some of the old beliefs need to be withdrawn on pain of inconsistency. Furthermore, there is typically more than one choice on which beliefs to give up.

The most influential attempt to model the belief revision process appeared in the early 1980's and is based on formal logic. In particular, beliefs are represented as logical sentences, and the process of belief revision is modelled as a function $*$ that maps a set of beliefs K (representing the agent's initial beliefs), and a sentence ϕ (representing the new information), to a new set of beliefs $K * \phi$. To capture the notion of *rational* belief change, a number of postulates were introduced by Alchourrón, Gärdenfors, and Makinson that regulate the revision function $*$; these postulates are now known as the *AGM postulates* for revision. It should be noted that AGM postulates define, not one, but a whole class of revision functions $*$, intuitively corresponding to different revision policies employed by different rational agents. In addition to the above axiomatic approach to Belief Revision, a number of *constructive models* have been proposed. Moreover important theorems (known as *representation results*) have been established that prove the equivalence of the axiomatic and constructive approaches.

In this talk we shall journey through the main models and results in Belief Revision. We shall also discuss the challenges that lie ahead, focusing mainly on *iterated revision* and *relevance*. Both are aspects of the revision process that have been left unattended by the AGM postulates. For iterated revision, one would like to capture the intuition that an agent's policy should, in a sense, be consistent thought-out a sequence of revisions. Likewise for relevance, we would like to formally encode the intuition that when revising a belief set K by new information ϕ, only the part of K that is *relevant* to ϕ may change; everything else should stay the same. We shall discuss recent progress on both these issues.

Konstantinos Tsaprounis (University of the Aegean)

On mathematical applications of large cardinal axioms

The various large cardinal axioms have been intensively studied during the last decades and have proven to be a very important and fruitful set-theoretic theme, with several mathematical applications. In this survey talk, we will start by giving a brief overview of the hierarchy of large cardinal axioms. Subsequently, we will concentrate on some specific notions and their properties, presenting in some more detail how their reflective nature has turned out to be useful in other mathematical contexts.

Nikos Tzevelekos (Queen Mary University of London)

Nominal game semantics

Game semantics has been developed since the 1990s as a denotational paradigm capturing observational equivalence in functional languages with imperative features. While initially introduced for PCF variants, the theory can nowadays express effectful languages ranging from ML fragments and Java programs to C-like code. In this talk we present recent advances in devising game models for effectful computation. Central in this approach is the use of names for representing in an abstract fashion different forms of notions and effects, such as references, higher-order values and polymorphism. We moreover look at automata models relevant to nominal games and how can they be used for model checking program equivalence.

Stathis Zachos (National Technical University of Athens)

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Contributed papers

Stamatis Dimopoulos (University of Bristol)

Strong compactness and the continuum function

Set theory is the branch of mathematical logic that studies the axiomatic system ZFC and its deductive strength. When ZFC is unable to decide the truth of a certain statement, large cardinals are used to "measure" how much more strength does ZFC need to decide it. One of the most interesting large cardinal notions is that of strong compactness, as there has been a large amount of work on the impact it has on the combinatorial properties of regular cardinals. However, strongly compact cardinals are not as flexible as other large cardinals when it comes to their interaction with forcing. For instance, it is still open whether it is possible to control the continuum function and at the same time preserve strong compactness, without relying on stronger properties such as supercompactness. In an ongoing work with A. Apter, we look at special cases of non-supercompact strongly compact cardinals and their preservation in forcing extensions with some control on the continuum function.

Initially, we show that assuming only a partial degree of supercompactness, it is possible to violate GCH at a non-supercompact strongly compact cardinal, while preserving the full extent of its strong compactness. Also, we show that certain Easton functions can be realised while preserving the strong compactness of the least measurable limit of supercompact cardinals. Finally, we show how to force a violation of GCH at all strongly compact cardinals, in models where strong compactness coincides with supercompactness.

Pavlos Marantidis (TU Dresden)

Approximate Unification in the Description Logic $\mathcal{FL}_0$

Description Logics are a well-investigated family of logic-based knowledge representation formalisms. They can be used to represent the relevant concepts of an application domain forming so-called ontologies. Unification in description logics has been introduced as a novel inference service that can be used to detect redundancies in ontologies, by finding different concepts that may potentially stand for the same intuitive notion. It was first investigated in detail for the description logic $\mathcal{FL}_0$, where unification can be reduced to solving certain language equations. In order to increase the recall of this method for finding redundancies, we introduce and investigate the notion of approximate unification, which basically finds pairs of concepts that "almost" unify. The meaning of "almost" is formalized using distance measures between concepts.

In this talk, we demonstrate how approximate unification in $\mathcal{FL}_0$ can be reduced to approximately solving language equations. The latter problem utilizes language distances and is of independent interest. We devise algorithms for two particular distances, that, interestingly enough, make use of many different tools from mathematics.

This is joint work with Franz Baader and Alexander Okhotin.